

# Research on the Comprehensive Energy Consumption of Aluminum in China based on Grey Model

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## Abstract

*There are serious problems of energy consumption among China's aluminum industry. To have a knowledge of the trend of comprehensive energy consumption of aluminum industry, we analyzed the data of comprehensive energy consumption of aluminum industry in China 2004-2013 in this paper based on the grey model GM (1,1). The result shows that the prediction accuracy class is A(excellent) by using the grey model for the comprehensive energy consumption of aluminum. The forecast results are reliable, and can reflect the actual situation and the trend of the comprehensive energy consumption of aluminum in China.*

**Keywords:** Aluminum; Comprehensive energy consumption; Grey model GM (1,1)

## INTRODUCTION

There are more than 50 years history of aluminum industry in China, and it has formed a relatively complete industrial cluster. Especially, the electrolytic aluminum and alumina industry is developing rapidly, and have a good development prospects. However, China's aluminum industry is facing development bottlenecks to be solved. For example, the comprehensive energy consumption of aluminum restricts the development of aluminum industry. So we will analyze the data of comprehensive energy consumption of aluminum industry of China in 2004-2013 in this paper based on the grey model GM (1, 1), and predict more accurate information about the comprehensive energy consumption of aluminum, so that we can provide more accurate data support for the macro energy decisions.

The grey system theory, which was founded by Deng Ju-long in 1982, is used to analysis those information which is not unknown neither clear. Depiction of the grey system method included the "gray equation", "grey matrix" and "grey parameters". It's commonly used to

forecast the energy consumption because it's less requirements for information, more simple for calculating and more accurate and easier testing. The GM (1,1)' accuracy depends on the average relative residuals ( $\epsilon_{avg}$ ) and the standard deviation ratio (C) and the small error probability(P). It's shown in the Table 1.

## DATA SOURCES

In this paper, the sample data of 2004-2013 are from the China Nonferrous Metals Industry Year Book. The simulation results are verified by the relative error test and residual test, and the precision of the model is analyzed. The data are shown in the Table 2.

According to the Table 2, the aluminum comprehensive energy consumption continued to decline year by year in 2004—2013. It shows that the energy efficiency of China's aluminum industry is gradually improving, the aluminum industry are more and more ecological and environmental protection. The trend is shown in figure 1.

## THE DIFFERENCE ANALYSIS

In this paper, GM (1, 1) is established by the comprehensive energy consumption of Aluminum and used to forecast the comprehensive energy consumption in the future. To simplify calculation, we used kwh/t as unified unit.

Let the aluminum composite power consumption sequence  $\chi(0)$ , then

$$\begin{aligned} \chi^{(0)} &= (\chi^{(0)}(1), \chi^{(0)}(2), \chi^{(0)}(3), \chi^{(0)}(4), \chi^{(0)}(5), \\ &\chi^{(0)}(6), \chi^{(0)}(7), \chi^{(0)}(8), \chi^{(0)}(9), \chi^{(0)}(10)) \\ &= (23328.4, 23063, 22517.6, 22310.5, \\ &21978.8, 22162.8, 21776.7, 21538.4, \\ &21340.9, 21057.4) \end{aligned}$$

Step one: Test and verify the feasibility of the model, carry out the ratio test.

The class ratio sequences of  $\chi^{(0)}$ :

$$\begin{aligned} \sigma^{(0)} &= (\sigma^{(0)}(1), \sigma^{(0)}(2), \sigma^{(0)}(3), \sigma^{(0)}(4), \\ &\sigma^{(0)}(5), \sigma^{(0)}(6), \sigma^{(0)}(7), \sigma^{(0)}(8), \sigma^{(0)}(9)) \\ &= (1.0115, 1.0242, 1.0093, 1.0151, \\ &0.9917, 1.0177, 1.0111, 1.0093, \\ &1.0135) \end{aligned}$$

All of the energy consumption data sequence  $\chi(0)$  is corresponding to the level ratio of the coverage area of  $\sigma^{(0)}(k) \in (e^{-\frac{2}{n+1}}, e^{\frac{2}{n+1}}) = (0.8338, 1.1994)$ , all the class ratio sequences are within the region, it is possible to establish GM (1,1) model.

Step two: accumulating a  $\chi(0)$ , generate a sequence  $\chi(1)$ .

$$\begin{aligned} \chi^{(1)} &= (\chi^{(1)}(1), \chi^{(1)}(2), \chi^{(1)}(3), \chi^{(1)}(4), \chi^{(1)}(5), \\ &\chi^{(1)}(6), \chi^{(1)}(7), \chi^{(1)}(8), \chi^{(1)}(9), \chi^{(1)}(10)) \\ &= (23328.4, 46391.4, 68909, 91219.5, \\ &113198.3, 135361.1, 157137.8, 178676.2, \\ &200017.1, 221074.5) \end{aligned}$$

Step three: calculate the estimating parameter columns.

Among them,

$$B = \begin{bmatrix} -\frac{1}{2}(\chi^{(1)}(1) + \chi^{(1)}(2)) & 1 \\ -\frac{1}{2}(\chi^{(1)}(2) + \chi^{(1)}(3)) & 1 \\ \vdots & \vdots \\ -\frac{1}{2}(\chi^{(1)}(n-1) + \chi^{(1)}(n)) & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -34859.9 & 1 \\ -57650.2 & 1 \\ -80064.25 & 1 \\ -102208.9 & 1 \\ -124279.7 & 1 \\ -146249.45 & 1 \\ -167907 & 1 \\ -189346.65 & 1 \\ -210545.8 & 1 \end{bmatrix},$$

$$Y_n = \begin{bmatrix} \chi^{(0)}(2) \\ \chi^{(0)}(3) \\ \vdots \\ \chi^{(0)}(n) \end{bmatrix} = \begin{bmatrix} 23063 \\ 22517.6 \\ 22310.5 \\ 21978.8 \\ 22162.8 \\ 21776.7 \\ 21538.4 \\ 21340.9 \\ 21057.4 \end{bmatrix}$$

After calculation,  $\hat{\alpha} = (\alpha, \beta)^T = (B^T B)^{-1} B^T Y_n =$

$$\begin{bmatrix} 0.010098153 \\ 23220.7192628 \end{bmatrix}$$

Among them,  $\alpha = 0.010098153$ ,  $\beta = 23220.7192628$

Step four: seek time response function that determines GM (1,1) forecast model.

$$\chi^{(1)}(0) = 23328.4, \frac{\beta}{\alpha} = 2299501.628$$

GM (1,1) time response function

$$\hat{\chi}^{(1)}(k+1) = \left[ \chi^{(1)}(0) - \frac{\beta}{\alpha} \right] e^{-\alpha k} + \frac{\beta}{\alpha}$$

,  $k=0, 1, 2, \dots, n$

So make sure GM (1,1) prediction model:

$$\begin{cases} \hat{\chi}^{(1)}(k+1) = -2276173.228e^{-0.010098153k} + 2299501.628 \\ \hat{\chi}^{(0)}(k+1) = \hat{\chi}^{(1)}(k+1) - \hat{\chi}^{(1)}(k) \quad (k=0, 1, 2, \dots) \end{cases}$$

Step five: GM (1,1) prediction model checking.

### (1) Residual test

According to the Equation, calculated  $\hat{\chi}^{(1)}(k)$ , and then ligo

$\hat{\chi}^{(0)}(k)$  Sequence, and calculate residuals  $\Delta^{(0)}(k) = \chi^{(0)}(k) - \hat{\chi}^{(0)}(k)$ , the relative

residuals  $= \varepsilon(k) = \frac{\Delta^{(0)}(k)}{\chi^{(0)}(k)} \times 100\%$  calculated

results are shown in Table 3.

The average residual of the model is

$$\varepsilon_{\text{avg}} = \frac{1}{n} \sum_{k=1}^n |\varepsilon(k)| = 0.4223\% < 0.01.$$

The prediction accuracy of the model is the primary (excellent) according to Table 1.

## (2) Posterior difference test

The mean and variance of the modeling sequence  $\chi^{(0)}(k)$ , respectively

$$\bar{\chi} = 22107.45, S_1^2 = 528475.4272, S_1 = 726.963154.$$

The mean and variance of residuals were

$$\bar{\Delta} = 0.17204, S_2^2 = 18656.66899, S_2 = 136.5894176.$$

Posterior difference ratio

$$C = \frac{S_2}{S_1} = 0.1867 < 0.35,$$

$$P = P\{|\Delta^{(0)}(k) - \bar{\Delta}^{(0)}| < 0.6745S_1\} = 1$$

$> 0.95$

After the test of the residuals and the posterior difference test, Comparison of test results with model accuracy test grade 1-1, the prediction accuracy of the GM (1,1) model in this paper is of the first order (excellent). The Specific fitting degree of the model is shown in figure 2.

Figure2 shows that the mathematical expression of the GM (1,1).It is specifically expressed as

$$\begin{cases} \hat{\chi}^{(1)}(k+1) = -2276173.228e^{-0.010098153k} + 2299501.628 \\ \hat{\chi}^{(0)}(k+1) = \hat{\chi}^{(1)}(k+1) - \hat{\chi}^{(1)}(k) \quad (k = 0, 1, 2, \dots) \end{cases}$$

It can be known that data fitting degree of the model is good, the forecast accuracy is high, and the actual data is basically consistent, so using the grey model can predict the comprehensive energy consumption of aluminum in the next few years.

## 4 Forecast Analysis

According to GM (1, 1) prediction equation:

$$\hat{\chi}^{(1)}(k+1) = -2276173.228e^{-0.010098153k} + 2299501.628$$

Respectively, make  $k=10,11,12\dots19$ . Get the forecast value of comprehensive energy consumption of aluminum in the next 10 years, the results are shown in Table 4.

Table 4 shows that the comprehensive energy consumption of aluminum in China will be decreased in the next ten year, it was 20883kwh/tin 2014, and then decreased to 19068 kwh/t in 2023, the average annual reduction rate is 8.69%. Compared with consumption in 2004-2013, the descend range was slightly slowed down, and the specific trend were shown in Figure 3.

## CONCLUSIONS

In this paper, the GM (1,1) model is used to predict the trend of the comprehensive energy consumption of aluminum in China in next ten years.

(1)The prediction accuracy class is A(excellent) by using the grey model for China's comprehensive energy consumption of aluminum .The forecast results are reliable, and reflect the actual situation and the trend of the comprehensive energy consumption of aluminum in China.

(2) Although the prediction of comprehensive energy consumption of aluminum in China shows a declining trend in the next 10 years, the production of aluminum industry in China should be adjusted as soon as possible to optimize the production process, and reduce consumption of energy.

(3) Compared with other mathematical model, GM (1,1) model is more simple and easier to be built, the requirement of regularity and integrity of the data is less. However, GM (1,1) model does not consider the long-term changes of economic environment and relevant national policies, as a result, GM (1,1) model is mainly limited to short-term and medium-term projections.

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**Table 1 GM (1,1)' accuracy class reference table**

Model accuracy class	The average relative residuals ( $\varepsilon_{avg}$ )	Standard deviation ratio C	Small error probability P
1st (excellent)	0.01	< 0.35	> 0.95
2nd (qualified)	0.05	< 0.5	> 0.8
3rd (marginal)	0.10	< 0.65	> 0.70
4th (defective)	0.20	$\geq 0.65$	$\leq 0.70$

**Table 2 Comprehensive energy consumption data**

Year	Comprehensive energy consumption of aluminum (Kwh/t)	Comprehensive energy consumption of alumina (Kgce/t)*	Comprehensive energy consumption of aluminum ingot (Kwh/t)
2004	23328.4	1180	15470
2005	23063	1130.5	15294
2006	22517.6	1090.2	14932.3
2007	22310.5	1023.4	14795
2008	21978.8	998.2	14575
2009	22162.8	802.7	14697
2010	21776.7	868.1	14441
2011	21538.4	794.4	14283
2012	21340.9	631.3	14152
2013	21057.4	590.6	13964

\* The power equivalent 0.1404kgce / kWh conversion

**Table3 Calculation results of the actual values, fitted values and residuals**

Years	$\chi^{(0)}(k)$	$\hat{\chi}^{(0)}(k)$	$\chi^{(1)}(k)$	$\hat{\chi}^{(1)}(k)$	$\Delta^{(0)}(k)$	$\varepsilon(k)\%$
2004	23328.4	23328.4	23328.4	23328.4	9.4587	4.0546
2005	23063	22869.48	46391.4	46197.88	193.5186	0.8390
2006	22517.6	22639.70	68909	68837.59	-122.1040	-0.5422
2007	22310.5	22412.24	91219.5	91249.82	-101.7353	-0.4559
2008	21978.8	22187.05	113198.3	113436.87	-208.2519	-0.9475
2009	22162.8	21964.13	135361.1	135401.01	198.6688	0.8964
2010	21776.7	21743.45	157137.8	157144.45	33.2498	0.1526
2011	21538.4	21524.99	178676.2	178669.44	13.4136	0.0622
2012	21340.9	21308.72	200017.1	199978.16	32.1824	0.1508
2013	21057.4	21094.62	221074.5	221072.78	-37.2216	-0.1767

**Table 4 Forecasting Result of aluminum in China**

Years	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023
Comprehensive energy consumption of aluminum	20883	20673	20466	20259	20056	19854	19655	19458	19262	19068



Figure 1 the trends of comprehensive energy consumption of aluminum

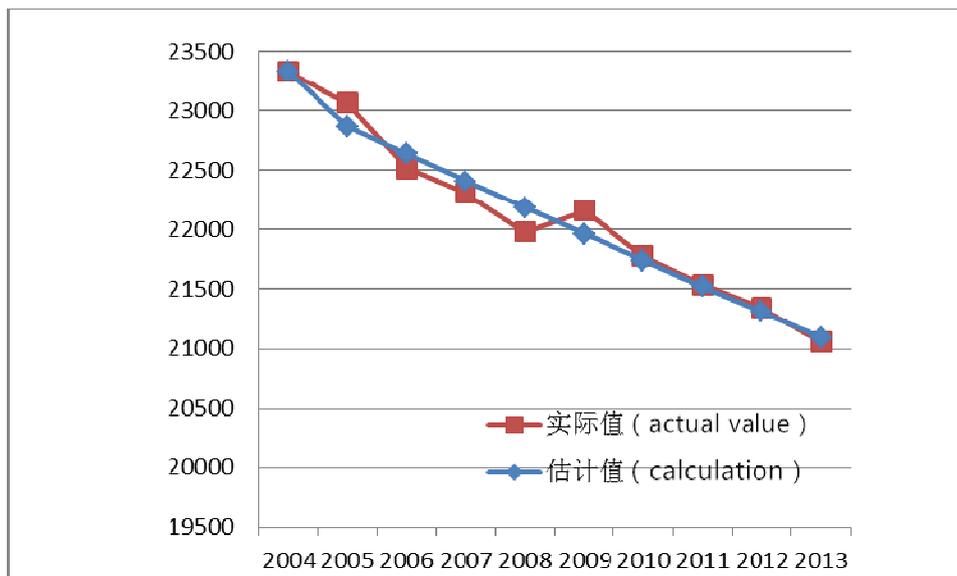
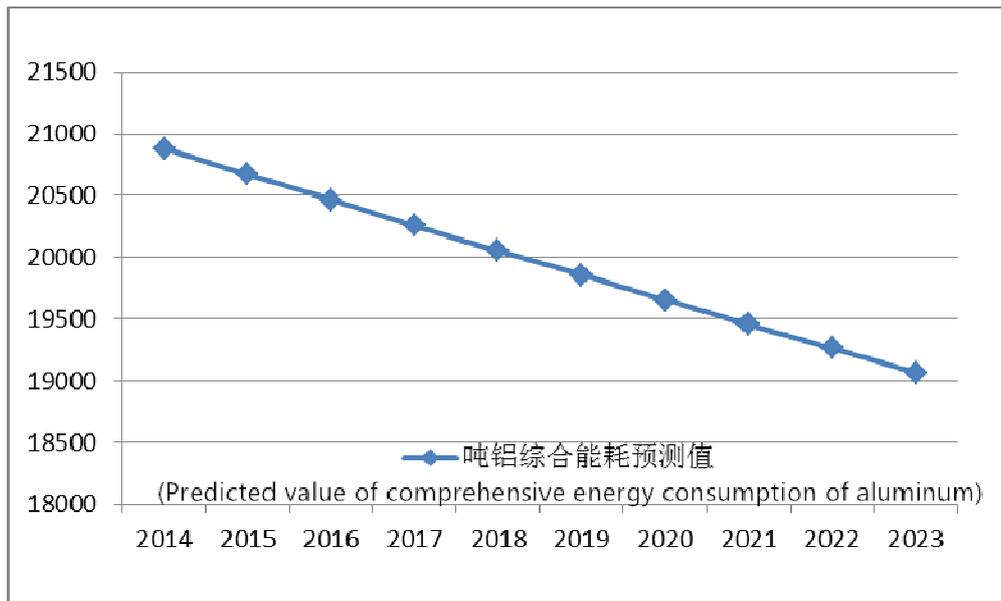


Figure 2 the fitting figure of calculation value and the actual value



**Figure 3 Forecast trend of comprehensive energy consumption of aluminum**